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Reg. No. : ....

Code No. : 20291 E **Sub. Code : AMMA 41**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Fourth Semester

Mathematics — Core

ABSTRACT ALGEBRA

(For those who joined in July 2020 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. Which of the following is not a symmetric relation on $S = \{a,b,c,d\}$?

 (a) $\{(a,b),(b,a)\}$

 (b) $\{(a,b),(b,c),(a,c)\}$

 (c) $\{(a,a),(b,b)\}$

 (d) $\{(a,b),(b,c),(b,a),(c,b)\}$

2. If the order of an element α is a group G is x then the order of the element α^{-1} is

 (a) -1 (b) $-x$

 (c) x (d) x^{-1}

3. In the group $G = \{1,-1,i,-i\}$ with usual multiplication, the inverse of i is ————

 (a) 1 (b) i

 (c) $-i$ (d) -1

4. Let G be a finite group and H be a subgroup of G. If $[G:H] = |G|$ then H is ————

 (a) $\{e\}$ (b) G

 (c) H (d) e

5. If $f: G \to G'$ is 1-1, then $O(\ker f) =$ ————

 (a) -1 (b) 0

 (c) 1 (d) 2

6. $R^+/\{1,-1\} \cong$ ————

 (a) R^+ (b) R^-

 (c) R (d) $\{1,-1\}$

7. In the ring $(R,+,\cdot)$ the set of units is ————

 (a) Z (b) $\{1,-1\}$

 (c) $R-\{0\}$ (d) R

8. In the ring $M_2(R)$, the unit element is

 (a) $\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$ (b) $\begin{pmatrix}0 & 0\\0 & 0\end{pmatrix}$

 (c) $\begin{pmatrix}2 & 4\\2 & 4\end{pmatrix}$ (d) $\begin{pmatrix}4 & 2\\2 & 4\end{pmatrix}$

9. Which one is a prime ideal in R?

 (a) (-1) (b) (0)

 (c) (1) (d) (2)

10. If $f(x), g(x) \in Z_4[x]$ be defined as $f(x) = x^2+3x+1$ and $g(x) = 2x^2+x$ then degree of $f(x).g(x)$ is ————

 (a) 3 (b) 4

 (c) 2 (d) 1

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Prove that the set of all equivalence classes determined by an equivalence relation defined on a set S forms a partition on the set S.

 Or

 (b) If $f: A \to B$, $g: B \to C$ are bijections, prove that $gof: A \to C$ is also a bijection.

12. (a) If G is a finite group with even number of elements then prove that G contains at least one element of order 2.

 Or

 (b) Let A and B be subgroups of a finite group G such that A is a subgroup of B. Show that $[G:A] = [G:B][B:A]$.

13. (a) Prove that every subgroup of a cyclic group is cyclic.

 Or

 (b) If $f: G \to G'$ is a group homomorphism prove that f is $1-1 \Leftrightarrow \ker f = \{e\}$.

14. (a) Prove that a finite commutative ring R without zero - divisors is a field.

Or

(b) Show that the only ideals of a field F are F and $\{0\}$.

15. (a) Show that z_n is an integral domain if and only if n is prime.

Or

(b) Prove that every finite integral domain is a field.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 600 words.

16. (a) Let A and B be two subgroups of a group G. Prove that AB is a subgroup of G if and only if $AB = BA$.

Or

(b) Prove that the union of two subgroups of a group G is a subgroup if and only if one is contained in the other.

17. (a) Let H and K be two finite subgroups of a group G. Prove that $|HK| = \frac{|H||K|}{|H \cap K|}$.

Or

(b) State and prove Lagrange's theorem.

18. (a) If $f : G \to G'$ is a homomorphism with Kernel K, prove that $\frac{G}{K} \cong f(G)$.

Or

(b) State and prove Cayley's theorem.

19. (a) Let R be a commutative ring with identity prove that an ideal M of R is a maximal ideal $\Leftrightarrow R/M$ is a field.

Or

(b) Prove the following

(i) z_n is an integral domain $\Leftrightarrow n$ is a prime number.

(ii) the characteristics of an integral domain is either 0 or a prime number.

20. (a) State and prove division algorithm.

Or

(b) Prove that every integral domain can be embedded in a field.